ELEMENTS
of the
DIFFERENTIAL and INTEGRAL CALCULUS.

DIFFERENTIAL CALCULUS.
Of the differentiation of algebraic quantities.

1. We say that one variable is a function of another when the first is equal to a certain compound analytical expression of the second; for example, \( y \) is a function of \( x \) in the following equations:
   \[ y = (a-x)^2, \quad y = x^2 - 3x^2, \quad y = \frac{x^2}{a}, \quad y = b + ox^2, \quad y = (a + bx + ax^2 + bx^3)^2. \]

2. Let us consider a function in its state of augmentation, in consequence of the increase of the variable which it contains. As every function of a variable \( x \) can be represented by the ordinate of a curve \( BMM \), let \( AB = x \) and \( TM = y \) be the ordinates of a point \( M \) of this curve, and let us suppose that the abscissa \( AB \) receives an increment \( PP = h \); the ordinate \( TM \) will become \( TM = y' \); Fig. 1. To obtain the value of this new ordinate, we see that it is necessary to change \( x \) into \( x + h \) in the equation of the curve, and the value which this equation will then determine for \( y \) will be that of \( y' \).

For example, if we had the equation \( y = mx^2 \), we should obtain \( y' \) by changing \( x \) into \( x + h \), and \( y \) into \( y' \); and we should have
   \[ y' = m(x+h)^2 = mx^2 + 2mh + mh^2. \]

3. Let us now take the equation \( y = x^2 \)
   \[ y' = (x+h)^2 = x^2 + 3xh + 3xh + h^2; \]
   if from this equation we subtract equation (1), there will remain
   \[ y' - y = 3xh + 3xh + h^2; \quad \text{and dividing by } h \]
   \[ \frac{y' - y}{h} = 3x + 3x + h. \]

Let us see what this result teaches us: \( y' - y \) represents the increment of the function \( y \) in consequence of the increment \( h \) given to \( x \); since this difference \( y' - y \) is that of the new state of magnitude of \( y \), as respects its primitive state.

On the other hand, the increment of \( x \) being \( h \), it follows that \( \frac{y' - y}{h} \), which is the ratio of the increment